

Supplementary Information

The cost of consistency: physics loss, data budget, and field fidelity in learned PDE solutions

Sehaj Randhir Singh

Correspondence: sehajrsinghs@gmail.com

SI 1. Full physics-vs-data sweep (12 runs)

Every run of the heat-plate sweep is committed in `physvdata_data.json`; the 12 rows are reproduced below (answer rel-MAE, field max/mean error, governing-equation residual).

Table 1: Physics-vs-data sweep, per run.

samples	w_{phys}	seed	answer	field max	field mean	residual
64	0.0	0	0.0215	0.116	0.024	2.60
64	0.0	1	0.0152	0.207	0.043	4.46
64	0.0	2	0.0245	0.150	0.027	4.04
64	0.05	0	0.0403	0.512	0.151	0.387
64	0.05	1	0.0231	0.496	0.138	0.370
64	0.05	2	0.0187	0.523	0.158	0.589
256	0.0	0	0.0110	0.032	0.007	0.815
256	0.0	1	0.0086	0.033	0.007	0.458
256	0.0	2	0.0125	0.032	0.007	0.347
256	0.05	0	0.0179	0.316	0.083	0.106
256	0.05	1	0.0121	0.270	0.076	0.079
256	0.05	2	0.0150	0.246	0.058	0.061

SI 2. Budget threshold

The 120-epoch model underfits the heat-plate field (peak error 48.5%, mean-field collapse); the 240-epoch model resolves the mode structure (canonical peak 5.9%, mean 1.3%). The 250-sample held-out evaluation has $R^2 = 0.998$ with mean relative error 0.020 (`fig10_data.json`). The canonical answer prediction is 308.0 vs. exact 300.0 (2.7%).

SI 3. DeepXDE baseline details

Per-instance PINNs (DeepXDE 1.15, PyTorch backend) trained on three Burgers instances: canonical ($\nu = 0.05$, $A = 1.5$, $\sigma = 0.3$), mild ($\nu = 0.1$, $A = 0.8$, $\sigma = 0.45$), shock ($\nu = 0.02$,

$A = 1.8$, $\sigma = 0.25$); 1116–1272 s of wall time each. The generalist is a single checkpoint scored by one forward pass per instance. All values in `deepxde_comparison.json`.

SI 4. Reproducibility

```
python3 physx/run_physvdata.py --stage score      # sweep statistics
python3 physx/run_fidelity_heat2d.py             # field fidelity scores
python3 -m unittest physx.test_physx             # 49 tests
```

The DeepXDE baseline reruns via the committed configuration in `physx/deepxde_baseline.py` (requires DeepXDE 1.15).